RF electromagnetic field and vortex penetration in multilayered superconductors

Takayuki Kubo,[1,*] Yoshihisa Iwashita,[2] and Takayuki Saeki[1]

[1] *Accelerator Laboratory, KEK, 1-1 Oho, Tsukuba, Ibaraki 305-0801 Japan*
[2] *Institute for Chemical Research, Kyoto University, Uji, Kyoto 611-0011 Japan*

Multilayered structure with a single superconductor layer and a single insulator layer deposited on a bulk superconductor is studied. General formulae for the vortex penetration-field of the superconductor layer and the magnetic field on the bulk superconductor which is shielded by the superconductor and insulator layers are derived with a rigorous calculation of the magnetic field attenuation in the multilayered structure. The formulae depend not only on the material and the thickness of the superconductor layer but also on the thickness of the insulator layer. The results can be applied to superconducting accelerating cavities with the multilayered structure. Using the formulae, a combination of the thicknesses of superconductor and insulator layers that can realize the enhanced rf breakdown field can be found for any given materials.

Technologies to fabricate the superconducting rf cavities made of Nb have been advanced. The maximum accelerating gradient $E_{\rm acc}$ of the TESLA type 1.3 GHz 9-cell cavities during performance tests in vertical cryostats regularly exceed 35 MV/m at several laboratories. The gradient record had been increasing and recently two 9-cell cavities made from large grain Nb reached 45 MV/m at DESY [1]. Further high gradients, however, would not be expected because their gradients are thought to be close to the empirical limit imposed by the thermodynamic critical field $\simeq 200\,{\rm mT}$ of Nb [2]. A. Gurevich suggested [3, 4] that a multilayered nanoscale coating on Nb cavity may push up the rf breakdown field to the level of the vortex penetration-field of the coating materials at which the Bean-Livingston surface barrier [5] disappears. While some experimental studies have been conducted on the subject based on the idea [6, 7], not much theoretical progress followed on it. In fact, the best parameter set for the multilayer coating model such as thicknesses of layers and choices of materials is not clear in the theoretical view. In this letter, the multilayered structure is carefully evaluated with a rigorous calculation on the electromagnetic field ditribution to keep its self-consistency. The resultant vortex penetration-field, the best combination of parameters, and materials are described.

The multilayer coating model [3] consists of alternating layers of superconductor layers ($\mathcal{S}$) and insulator layers ($\mathcal{I}$). The simplest configuration with a single superconductor layer and a single insulator layer is seen in Fig. 1. Each $\mathcal{S}$ layer is expected to withstand higher field than bulk Nb, and to shield the bulk Nb from the applied rf surface field B_0, because B_i (an rf surface field on the bulk Nb) is smaller than B_0. Then the multilayered structure is thought to withstand a higher field than the bulk Nb if B_0 is smaller than the vortex penetration-fields of the top $\mathcal{S}$ layer and B_i is smaller than that of the bulk Nb. The vortex penetration-field of the $\mathcal{S}$ layer was given by $B_v = \phi_0/4\pi\lambda\xi$ in the original paper [3], where $\phi_0 = 2.07\times 10^{-15}\,{\rm Wb}$ is the flux quantum [8], and λ and ξ are a London penetration depth and a coherence length of the material of the $\mathcal{S}$ layer, respectively. This expression, however, has the same form as the vortex penetration-field of the semi-infinite superconductor, and does not depend on any parameters on the configuration of the multilayered structure such as the $\mathcal{S}$ layer thickness or the $\mathcal{I}$ layer thickness. In order to incorporate effects from the configuration of multilayered structure, we carried out rigorous caluculation on the distribution of magnetic field and Meissner current in the $\mathcal{S}$ layer.

In order to derive the electromagnetic field in the multilayered structure, the Maxwell equations and the London equations should be solved with appropriate boundary conditions simultaneously. Contributions to the electromagnetic field distribution from the normal (unpaired) electrons of the superconductor and dielectric losses in the insulator are neglected. For simplicity, let us consider a model with a single $\mathcal{S}$ layer and a single $\mathcal{I}$ layer deposited on a bulk superconductor as shown in Fig. 1. Table. I shows the parameters for the model. $d_{\mathcal{I}}$ is assumed to be zero or larger than a few nm to suppress the Josephson coupling [4]. All layers are parallel to the y-z plane and then perpendicular to the x-axis. The applied electric and magnetic field are assumed to be parallel to the layers. Further we assume the materials used for the $\mathcal{S}$ layer is extreme Type II superconductor $\lambda_1 \gg \xi_1$, and the $\mathcal{S}$ layer thickness is larger than the the coherence length $d_{\mathcal{S}} \gg \xi_1$. Note that the $\mathcal{S}$ layer of our model is not necessarily a thin film hence the discussion below can be applied to any $\mathcal{S}$ layer with arbitrary thickness $d_{\mathcal{S}} \gg \xi_1$. Solving the Maxwell equations in the $\mathcal{I}$ layers, and the Maxwell-London equations in the $\mathcal{S}$ layers and in the bulk superconductor, we find

$$B_{\rm I} = B_0 \frac{\lambda_1 \cosh\frac{d_{\mathcal{S}}-x}{\lambda_1} + (\lambda_2 + d_{\mathcal{I}})\sinh\frac{d_{\mathcal{S}}-x}{\lambda_1}}{\lambda_1 \cosh\frac{d_{\mathcal{S}}}{\lambda_1} + (\lambda_2 + d_{\mathcal{I}})\sinh\frac{d_{\mathcal{S}}}{\lambda_1}}\,, \qquad (1)$$

$$B_{\rm II} = B_0 \frac{\lambda_1}{\lambda_1 \cosh\frac{d_{\mathcal{S}}}{\lambda_1} + (\lambda_2 + d_{\mathcal{I}})\sinh\frac{d_{\mathcal{S}}}{\lambda_1}}\,, \qquad (2)$$

$$B_{\rm III} = B_0 \frac{\lambda_1 e^{-\frac{x-d_{\mathcal{S}}-d_{\mathcal{I}}}{\lambda_2}}}{\lambda_1 \cosh\frac{d_{\mathcal{S}}}{\lambda_1} + (\lambda_2 + d_{\mathcal{I}})\sinh\frac{d_{\mathcal{S}}}{\lambda_1}}\,, \qquad (3)$$

* kubotaka@post.kek.jp

TABLE I. Parameters of the multilayered structure with a single superconductor layer and a single insulator layer deposited on a bulk superconductor.

Region	Material type	Parameter
I	Superconductor layer	Coherence length: ξ_1, London penetration depth: λ_1 ($\gg \xi_1$), Thickness: $d_{\mathcal{S}}$ ($\gg \xi_1$)
II	Insulator layer	Relative permittivity: ϵ_r, Thickness: $d_{\mathcal{I}}$ (zero or larger than a few nm)
III	Bulk superconductor	London penetration depth: λ_2

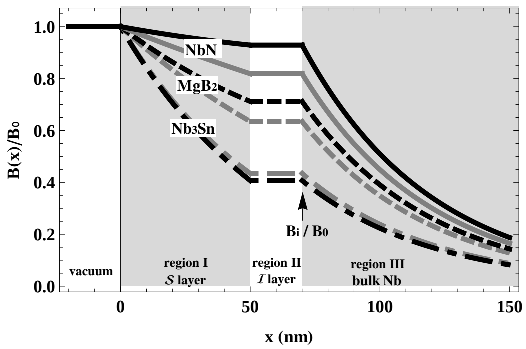

FIG. 1. Examples of the magnetic field attenuations in the multilayered structure. Black lines show our formulae given above, and gray lines show the naive estimates with $B = B_0 e^{-x/\lambda_1}$. Solid lines, dashed lines and dashed-dotted lines correspond to the material of the $\mathcal{S}$ layer: NbN ($\lambda_1 = \lambda_{\rm NbN} = 250\,{\rm nm}$), MgB_2 ($\lambda_1 = \lambda_{{\rm MgB}_2} = 110\,{\rm nm}$) and Nb_3Sn ($\lambda_1 = \lambda_{{\rm Nb_3Sn}} = 60\,{\rm nm}$), respectively. The bulk superconductor is assumed to be Nb ($\lambda_2 = \lambda_{\rm Nb} = 50\,{\rm nm}$). The thickness of the $\mathcal{S}$ layer and the $\mathcal{I}$ layer are fixed at $d_{\mathcal{S}} = 50\,{\rm nm}$ and $d_{\mathcal{I}} = 20\,{\rm nm}$.

where $B_{\rm I}$, $B_{\rm II}$, and $B_{\rm III}$ are magnetic fields in region I, II, and III, respectively. Note here that these equations are valid for $d_{\mathcal{I}} \ll (\sqrt{\epsilon_r}k)^{-1} \simeq 1\,{\rm cm}$, where $k = \omega/c$, $\omega = 2\pi \times 1.3 \times 10^9\,{\rm s}^{-1}$ is the angular frequency and c is the speed of light. Since we consider an insulator thickness $d_{\mathcal{I}} \ll 1\,{\rm cm}$ below, Eq. (1), (2) and (3) are accurate enough. It is easy to confirm that these equations are reduced to the well known expression for the semi-infinite superconductor given by $B = B_0 e^{-x/\lambda_1}$ when the $\mathcal{S}$ layer and the bulk superconductor are the same material ($\lambda_1 = \lambda_2$) and the $\mathcal{I}$ layer vanishes ($d_{\mathcal{I}} \to 0$). Fig. 1 shows examples how a magnetic field attenuates in a multilayered structure.

The vortex penetration-field can be evaluated by computing two forces acting on a vortex at a top of the $\mathcal{S}$ layer: a force from an image current of an image vortex which is introduced to satisfy a boundary condition of zero current normal to the surface, and another from a Meissner current $j_{\rm M}$ due to existence of external field which can be computed from Eq. (1) with $j_{{\rm M}y} = -(1/\mu_0) dB_{\rm I}/dx$. Then the vortex penetration field is given by

$$B_v = \frac{\phi_0}{4\pi\lambda_1\xi_1} \frac{\lambda_1 \cosh\frac{d_{\mathcal{S}}}{\lambda_1} + (\lambda_2 + d_{\mathcal{I}}) \sinh\frac{d_{\mathcal{S}}}{\lambda_1}}{\lambda_1 \sinh\frac{d_{\mathcal{S}}}{\lambda_1} + (\lambda_2 + d_{\mathcal{I}}) \cosh\frac{d_{\mathcal{S}}}{\lambda_1}}, \qquad (4)$$

which depends on both the $\mathcal{S}$ layer thickness $d_{\mathcal{S}}$ and the $\mathcal{I}$ layer thickness $d_{\mathcal{I}}$. Note here that Eq. (4) is reduced to the well-known expression $\phi_0/4\pi\lambda_1\xi_1$ for the semi-infinite $\mathcal{S}$ layer ($d_{\mathcal{S}} \to \infty$). As is obvious from Eq. (4), the vortex penetration-field decreases as the $\mathcal{I}$ layer thickness $d_{\mathcal{I}}$ increases. This behavior can be understood from the above results that the magnetic field attenuates more rapidly in an $\mathcal{S}$ layer on a thick $\mathcal{I}$ layer than that on a thin $\mathcal{I}$ layer. This means that a Meissner current, which is proportional to a gradient of the magentic field, becomes larger in an $\mathcal{S}$ layer on a thick $\mathcal{I}$ layer than that on a thin $\mathcal{I}$ layer. As a result the vortex is strongly drawn into the $\mathcal{S}$ layer by the force from the Meissner current.

Let us define the magnetic field attenuation ratio α by $\alpha = B_{\rm II}/B_0$. Then the shielded magnetic field $B_{\rm II}$ on the bulk Nb is given by αB_v when the applied magnetic field B_0 equals B_v. When the magnetic field attenuation in the $\mathcal{S}$ layer is enough for the shielded magnetic field αB_v to become smaller than 200 mT, which is thought to be the maximum field for the bulk Nb, the bulk Nb is safely protected. Then the maximum peak surface-field $B_{\rm max}$ is given by B_v. On the other hand when the magnetic field attenuation is not enough and αB_v is larger than 200 mT, $B_{\rm max}$ is limited by $\alpha^{-1} \times 200\,{\rm mT}$. Thus we find

$$B_{\rm max} = \begin{cases} B_v & (\alpha B_v < 200\,{\rm mT}) \\ \alpha^{-1} \times 200\,{\rm mT} & (\alpha B_v \geq 200\,{\rm mT}). \end{cases} \qquad (5)$$

Fig. 2 shows contour plots of $B_{\rm max}$. In order to improve $B_{\rm max}$, appropriate parameter regions should be chosen, otherwise $B_{\rm max}$ becomes smaller than that of the bulk Nb. As is obvious from comparison between two contour plots, the material of the $\mathcal{S}$ layer drastically changes the effect of the multilayered structure. As shown in Fig. 2(a), only small area of the parameter space is allowed for the enhancement of $B_{\rm max}$ with a single NbN layer. In addition, even if the best parameter set are chosen, the multilayered structure with a single NbN layer hardly can enhance $B_{\rm max}$. On the other hand, as shown in Fig. 2(b), Nb_3Sn layer significantly increases $B_{\rm max}$ in large area of the parameter space because of its superior screening effect and its high vortex penetration-field.

The general formulae of magnetic field distributions on structure of a couple of superconductor and insulator layers on a bulk superconductor are solved self-consistently. Vortex penetration-field for a superconductor sheet with different field level on both the sides is also derived. Combining these results, we can survey optimum combination of thicknesses on $\mathcal{S}$ and $\mathcal{I}$ layers, and $\mathcal{S}$ layer material.

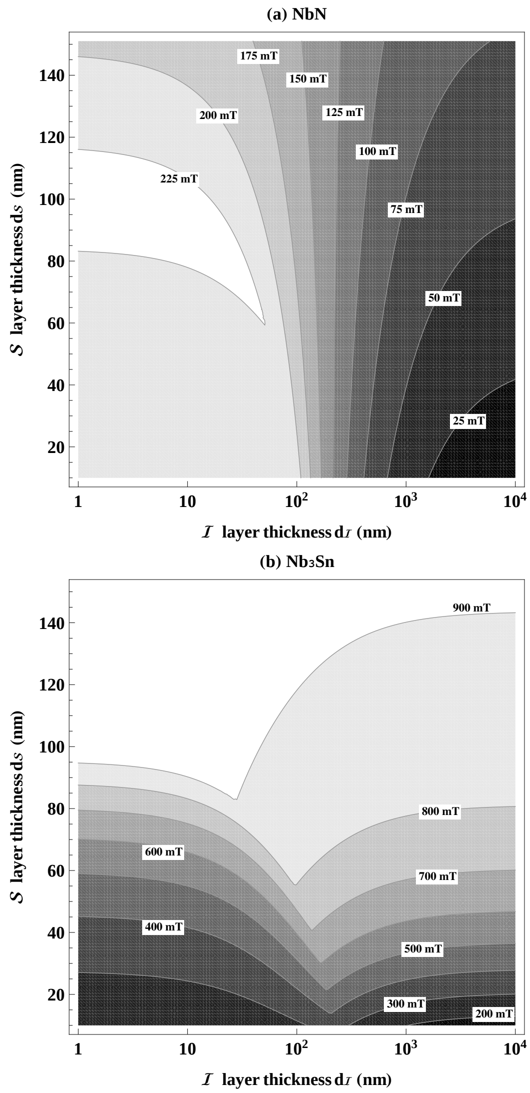

FIG. 2. Contour plots of the maximum peak surface-field $B_{\rm max}$. The abscissa represents the $\mathcal{I}$ layer thickness $d_{\mathcal{I}}$ and the ordinate represents the $\mathcal{S}$ layer thickness $d_{\mathcal{S}}$. Values written in the plot area are $B_{\rm max}$ in the unit of mT. The top and bottom figures correspond to materials of the $\mathcal{S}$ layer: (a) NbN ($\lambda_1 = \lambda_{\rm NbN} = 250\,{\rm nm}$, $\xi_1 = \xi_{\rm NbN} = 5\,{\rm nm}$) and (b) $\rm Nb_3Sn$ ($\lambda_1 = \lambda_{\rm Nb_3Sn} = 60\,{\rm nm}$, $\xi_1 = \xi_{\rm Nb_3Sn} = 3\,{\rm nm}$), respectively. The bulk superconductor is assumed to be Nb ($\lambda_2 = \lambda_{\rm Nb} = 50\,{\rm nm}$).

These procedure shows how the alternative layered structure can contribute to raise the maximum rf surface field. Hence it is possible for us to evaluate how the introduction of this structure on the inner-surface of superconducting cavity improves its achievable accelerating gradient.

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